

Solving the coincidence problem in a large class of running vacuum cosmologies

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Decaying vacuum cosmological models evolving smoothly between two extreme (very early and late time) de Sitter phases are capable to solve naturally several cosmic problems, among them: (i) the singularity, (ii) the horizon, (iii) the graceful-exit from inflation. Here we discuss a solution the coincidence problem based on a large class of running vacuum cosmologies evolving from de Sitter to de Sitter recently proposed. It is argued that even the cosmological constant problem can be solved provided that the characteristic scales of the limiting de Sitter manifolds are predicted from first principles.

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Current high quality astronomical observations are being successfully explained by the present cosmic paradigmatic scenario (Λ CDM plus inflation). However, the so-called cosmic concordance model can hardly provide, by itself, a definite explanation for the complete cosmic evolution involving two unconnected accelerating inflationary regimes separated by many aeons. Unsolved problems include the predicted existence of a spacetime singularity in the begin of the Universe, the “graceful-exit” from primordial inflation and the coincidence problem. Last but not the least, the scenario is also plagued with the so-called cosmological constant problem [1].

One possibility to solve such evolutionary problems is to incorporate energy transfer among the cosmic components, as happens in decaying $\Lambda(t)$ -models (running vacuum cosmologies, $\rho_\Lambda \equiv \Lambda(t)/8\pi G$) or, more generally, in the interacting dark energy cosmologies. Here we are interested in the first class of models because the idea of a time-varying vacuum energy density in the expanding Universe is physically more plausible than the current view of a strict constant Λ [2–6].

Recently, a large class of flat nonsingular FRW type cosmologies, where the vacuum energy density evolves like a truncated power-series in the Hubble parameter H , has been discussed in the literature [7, 8] (its dominant term behaves like $\rho_\Lambda(H) \propto H^{n+2}, n > 0$). Such models has some interesting features, among them: (i) a new mechanism for inflation with no “graceful-exit” problem,

(ii) the late-time expansion history is very close to the cosmic concordance model, and (iii) a smooth link between the initial and final de Sitter stages through the radiation and matter dominated phases.

In this letter we will show in detail how the coincidence problem is solved in this framework. We also advocate here that a generic running vacuum cosmology providing a complete cosmic history evolving between two extreme de Sitter phases are potentially able to solve the main cosmological problems.

The Einstein equations, $G^{\mu\nu} = 8\pi G [T_{(\Lambda)}^{\mu\nu} + T_{(T)}^{\mu\nu}]$, for the vacuum-matter interacting mixture read [7, 8]:

$$8\pi G \rho_T + \Lambda(H) = 3H^2, \quad (1)$$

$$8\pi G p_T - \Lambda(H) = -2\dot{H} - 3H^2, \quad (2)$$

where $\rho_T = \rho_M + \rho_R$ and $p = p_M + p_R$ are the total energy density and pressure of the material medium formed by nonrelativistic matter and radiation. Note that the bare Λ appearing in the geometric side was absorbed on the matter-energy side in order to describe the effective vacuum with energy density $\rho_\Lambda = -p_\Lambda \equiv \Lambda(H)/8\pi G$. Naturally, the time dependence of Λ is provoked by the vacuum energy transfer to the fluid component. In this context, the total energy conservation law, $u_\mu [T_{(\Lambda)}^{\mu\nu} + T_{(T)}^{\mu\nu}]_{;\nu} = 0$, assumes the following form:

$$\dot{\rho}_T + 3H(\rho_T + p_T) = -\dot{\rho}_\Lambda \equiv -\frac{\dot{\Lambda}}{8\pi G}. \quad (3)$$

What about the behavior of $\dot{\Lambda}$? Assuming that the created particles have zero chemical potential and that the vacuum fluid behaves like a condensate carrying no entropy, as happens in the Landau-Tisza two-fluid description employed in helium superfluid dynamics[9], it has

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been show that $\dot{\Lambda} < 0$ as a consequence of the second law of thermodynamics [10], that is, the vacuum energy density diminishes in the course of the evolution. Therefore, in what follows we consider that the coupled vacuum is continuously transferring energy to the dominant component (radiation or nonrelativistic matter components). Such a property defines precisely the physical meaning of decaying or running vacuum cosmologies in this work.

Now, by combining the above field equation it is readily checked that:

$$\dot{H} + \frac{3(1+\omega)}{2}H^2 - \frac{1+\omega}{2}\Lambda(H) = 0, \quad (4)$$

where the equation of state $p_T = \omega\rho_T$ ($\omega \geq 0$) was used. The above equations are solvable only if we know the functional form of $\Lambda(H)$.

The decaying vacuum law adopted here was first proposed based on phenomenological grounds [4, 6, 11–13], and latter on suggested by the renormalization group approach techniques applied to quantum field theories in curved spacetimes [14]. It is given by:

$$\Lambda(H) \equiv 8\pi G\rho_\Lambda = c_0 + 3\nu H^2 + \alpha \frac{H^{n+2}}{H_I^n}, \quad (5)$$

where H_I is an arbitrary time scale describing the primordial de Sitter era (the upper limit of the Hubble parameter), ν and α are dimensionless constants, and c_0 is a constant with dimension of $[H]^2$.

In a point of fact, the constant α above does not represent a new degree of freedom. It can be determined with the proviso that for large values of H , the model starts from a de Sitter phase with $\rho = 0$ and $\Lambda_I = 3H_I^2$. In this case, from (5) one finds $\alpha = 3(1 - \nu)$ because the first two terms there are negligible in this limit [see Eq. (1) in [11] for the case $n = 1$ and [13] for a general n]. The constant c_0 can be fixed by the time scale of the final de Sitter phase. For $H \ll H_I$ we also see from (4) that $c_0 = 3(1 - \nu)H_F^2$, where H_F characterizes the final de Sitter stage (see Eqs. (6) and (8)). Hence, the phenomenological law (5) assumes the final form:

$$\Lambda(H) = 3(1 - \nu)H_F^2 + 3\nu H^2 + 3(1 - \nu)\frac{H^{n+2}}{H_I^n}. \quad (6)$$

This is an interesting 3-parametric phenomenological expression. It depends on the arbitrary dimensionless constant ν and also of the two extreme Hubble parameters (H_I , H_F) describing the primordial and late time inflationary phases, respectively. Current observations imply that the value of ν is very small, $|\nu| \sim 10^{-6} - 10^{-3}$ [15]. More interesting, the analytical results discussed below remain valid even for $\nu = 0$. In this case, we obtain a sort of minimal model defined only by a pair of physical time scales, H_I and H_F , determining the entire evolution of the Universe. As we shall see, the possible existence of

these extreme de Sitter regimes suggest a different perspective to the cosmological constant problem.

By inserting the above expression into (3) we obtain the equation of motion:

$$\dot{H} + \frac{3(1+\omega)(1-\nu)}{2}H^2 \left[1 - \frac{H_f^2}{H^2} - \frac{H^n}{H_I^n} \right] = 0. \quad (7)$$

In principle, all possible de Sitter phases here are simply characterized by a constant Hubble parameter (H_c) satisfying the conditions $\dot{H} = \rho = p = 0$ and $\Lambda = 3H_c^2$. For all physically relevant values of ν and ω in the present context, we see that the condition $\dot{H} = 0$ is satisfied whether the values of H_c are constrained by the algebraic equation involving the arbitrary (initial and final) de Sitter vacuum scales H_I and H_F :

$$H_c^{n+2} - H_I^n H_c^2 + H_F^2 H_I^n = 0. \quad (8)$$

In particular, for $n = 2$, the value preferred from the covariance of the action, the exact solution is given by:

$$H_c^2 = \frac{H_I^2}{2} \pm \frac{H_I^2}{2} \sqrt{1 - 4H_F^2/H_I^2}, \quad (9)$$

and since $H_F \ll H_I$ we see that the two extreme scaling solutions for $n = 2$ are $H_{1c} = H_I$ and $H_{2c} = H_F$. However, we also see directly from (8) that the condition $H_F \ll H_I$, also guarantees that such solutions are valid regardless of the values of n . In certain sense, since H_0 is only the present day expansion rate, characterizing a quite casual stage of the recent evolving Universe, probably, it is not the interesting scale to be a priori predicted. In what follows we consider that the pair of extreme de Sitter scales (H_I , H_F) are the physically relevant quantities. This occurs because different from H_0 , the expanding de Sitter rates are associated with very specific limiting manifolds. For instance, de Sitter spaces are static when written in a suitable coordinate system. One idea to be advocated here is that the prediction of such scales, at least in principle, should be an interesting theoretical target. Their first principles prediction would open a new and interesting route to investigate the cosmological constant problem.

The solutions for the Hubble parameter describing analytically the transitions vacuum-radiation ($\omega = 1/3$) and matter-vacuum ($\omega = 0$) can be expressed in terms of the scale factor, the couple of scales (H_I , H_F) and free parameters (ν , n):

$$H = \frac{H_I}{[1 + Ca^{2n(1-\nu)}]^{1/n}}, \quad (10)$$

$$H = H_F [Da^{-3(1-\nu)} + 1]^{1/2}. \quad (11)$$

We remark that the transition radiation-matter is like in the standard cosmic concordance model. The only

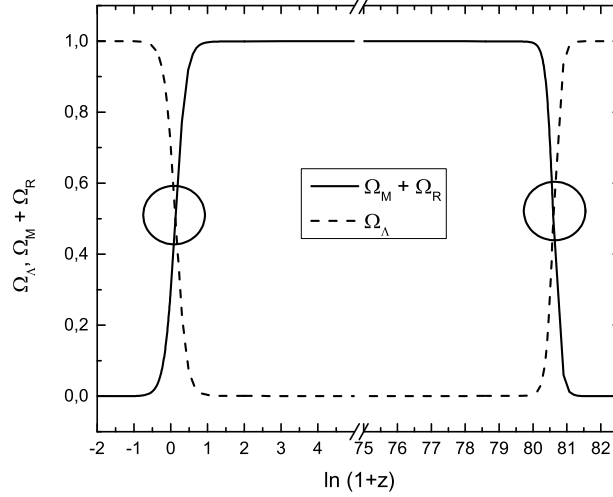


FIG. 1: Solution of the coincidence problem in running vacuum cosmologies. Solid and dashed lines represent the evolution of the vacuum (Ω_Λ) and total matter-radiation ($\Omega_M + \Omega_R$) density parameters for $n=2$, $\nu = 10^{-3}$ and $H_I/H_0 = 10^{60}$. Note that the values 5 and 75 in the horizontal axis were glued in order to show the complete evolution (the suppressed part present exactly the same behavior). Different values of n changes slightly the value of the redshift for which $\Omega_\Lambda = \Omega_M + \Omega_R$ at the very early Universe (see also discussion in the text).

difference is due to the small ν parameter that can be fixed to be zero (minimal model). Indeed, if one fixes $\nu = 0$, the matter-vacuum transition is exactly the same one of appearing of the flat Λ CDM model. As we shall see below, the scale H_F is a simple function of H_0 , ν and Ω_Λ .

The characteristic scales present in the above solutions specify the evolution during the de Sitter phases: the primordial vacuum solution with $Ca^{2n(1-\nu)} \ll 1$ and $H = H_I$, behaves like a “repeller” in the distant past, while the final vacuum solution for $a \gg 1$, that is, $Da^{3(1-\nu)} \mapsto 0$ and $H = H_F$ is an attractor in the distant future. The arbitrary constants C and D are easily determined. C can be fixed by the end of the primordial inflation ($\ddot{a} = 0$) or equivalently $\rho_\Lambda = \rho_R$. This means that $C = a_{(eq)}^{-2n(1-\nu)}/(1-2\nu)$ [$a_{(eq)}$ corresponds to the value of the scale factor at vacuum-radiation equality]. In terms of the present day observable quantities we also find $D = \Omega_{M0}/(\Omega_{\Lambda0} - \nu)$ and $H_F = H_0\sqrt{\Omega_{\Lambda0} - \nu}/\sqrt{1-\nu}$. For $\nu = 0$ and $\Omega_{\Lambda0} \sim 0.7$ one finds $H_F \sim 0.83H_0$, as expected a little smaller than H_0 . The small observable parameter $\nu < 10^{-3}$ quantifies the difference between the late time decaying vacuum model and the cosmic concordance cosmology, namely:

$$H = \frac{H_0}{\sqrt{1-\nu}} [\Omega_{M0}a^{-3(1-\nu)} + 1 - \Omega_{M0} - \nu]^{1/2}. \quad (12)$$

As remarked above, the $H(a)$ expression of the standard Λ CDM model is fully recovered for $\nu = 0$.

The solution of the coincidence problem in the present framework can be demonstrated as follows. The density

parameters of the vacuum and matter are given by:

$$\Omega_\Lambda \equiv \frac{\Lambda(H)}{3H^2} = \nu + (1-\nu)\frac{H_F^2}{H^2} + (1-\nu)\frac{H^n}{H_I^n}, \quad (13)$$

$$\Omega_T \equiv 1 - \Omega_\Lambda = 1 - \nu - (1-\nu)\frac{H_F^2}{H^2} - (1-\nu)\frac{H^n}{H_I^n}. \quad (14)$$

Such results are a simple consequence of expression (6) for $\Lambda(H)$ and the constraint Friedman equation (1). Note that $\Omega_T \equiv \Omega_M + \Omega_R$ is always describing the dominant component, either the nonrelativistic matter ($\omega = 0$) or radiation ($\omega = 1/3$).

The density parameters of the vacuum and material medium are equal in two different epochs specifying the dynamic transition between different dominant components. These different moments of time will be characterized here by Hubble parameters H_1^{eq} and H_2^{eq} . The first equality (vacuum-radiation, $\rho_\Lambda = \rho_R$) occurs just at the end of the first accelerating stage ($\ddot{a} = 0$), that is, when $H_1^{eq} = (\frac{1-2\nu}{2(1-\nu)})^{1/n} H_I$, while the second one is at low redshifts when $H_2^{eq} = (\frac{2(1-\nu)}{1-2\nu})^{1/2} H_F$. Note that such results are also valid for the minimal model by taking $\nu = 0$. In particular, inserting $\nu = 0$ in the first expression above we find $H_1^{eq} = H_I/2^{1/n}$. By adding the result $H_F = 0.83H_0$ we also find for $\nu = 0$ that $H_2^{eq} \sim 1.18H_0$, which is higher than H_0 , as should be expected for the matter-vacuum transition.

Naturally, the existence of two subsequent equalities on the density parameter suggests a solution to the coincidence problem. Neglecting terms of the order of 10^{-120}

and 10^{-60n} , it is easy to demonstrate the following results:

- 1) $\lim_{H \rightarrow H_I} \Omega_\Lambda = 1$ and $\lim_{H \rightarrow H_I} \Omega_M = 0$,
- 2) $\lim_{H \rightarrow H_F} \Omega_\Lambda = 1$ and $\lim_{H \rightarrow H_F} \Omega_M = 0$.

The meaning of the above results is quite clear. The density parameters of the vacuum and material components (radiation + matter) perform a cycle, that is, Ω_Λ , and $\Omega_M + \Omega_R$ are periodic in the long run.

In Figure 1, we show the complete evolution of the vacuum and matter-energy density parameters. Note the cyclic values of Ω_Λ and $\Omega_M + \Omega_R$. These parameters start and finish the evolution satisfying the above limits. The physical meaning is also remarkable. The model starts as a pure unstable vacuum de Sitter phase with $H = H_I$ (in the begin there is no matter or radiation), and evolves smoothly to a quasi-radiation phase parametrized by the small ν -parameter. For $\nu = 0$ the primordial nonsingular vacuum states deflates directly to the standard FRW radiation phase.

Later on, also occurs the transition from radiation-vacuum to matter-vacuum dominated phase. The “irreversible entropic cycle” from de initial Sitter (H_I) to the late time de Sitter stage is completed when the Hubble parameter approaches its extremely small final value ($H \mapsto H_F$). The de Sitter spacetime that was a ‘repeller’ (unstable solution) at very early times ($z \rightarrow \infty$) it will become an attractor in the distant future ($z \rightarrow -1$) determined by the incredibly low energy scale H_F .

Like the above solution to the coincidence problem, some cosmological puzzles can also be resolved along the same lines because the time behavior of the present scenario even fixing $\alpha = 1 - \nu$ has been proven here to be exactly the one discussed in Ref. [8] (see also [11] for the case $n = 1$). At this point, in order to comment and compare these results with alternative models also evolving between two extreme de Sitter stages, it is interesting to review briefly the possible solution for the main cosmological problems for this class of models driven by a decaying vacuum state:

- *Singularity*: The spacetime in the distant past is a nonsingular de Sitter geometry with an arbitrary energy scale H_I . This means that the horizon problem is naturally solved. In order to agree with the semi-classical description of gravity, the arbitrary scale H_I must be constrained by the lower limit $H_I^{-1} \geq 10^{-43} \text{sec}$ (Planck time), or equivalently, $H_I \leq 10^{19} \text{ GeV}$ (Planck energy) in natural units.
- *“Graceful-Exit” Problem*: The transition from the early de Sitter to the radiation phase is smooth and driven by Eq. (10). The first coincidence of density parameters happens for $H = H_1^{eq}$, $\rho_\Lambda = \rho_R$ and $\ddot{a} = 0$, that is, when the first inflationary period ends. All the radiation entropy ($S_0 \sim 10^{88}$, in dimensionless units) and matter-radiation content now observed was generated during the early decaying vacuum process (see [16] for the entropy

produced for $n = 2$. The general case will be presented elsewhere[17]). Some possible curvature effects has already been considered in [12, 18].

- *Cosmological Constant Problem*: It is widely known that phenomenological decaying vacuum models are unable to solve this conundrum [1, 3–6]. The basic reason seems to be related with the clear impossibility to predict the present day value of the vacuum energy density (or equivalently the value of H_0) from first principles. However, a new phenomenological approach can provide an expected line of inquire in the search for alternative (first principle) solutions for this remarkable puzzle. In this concern, we notice that the minimal model discussed here depends only on two relevant physical scales (H_F, H_I) which are associated to the extreme de Sitter phases. The existence of such scales implies that the ratio between the late and very early vacuum energy densities $\rho_{\Lambda F}/\rho_{\Lambda I} = (H_F/H_I)^2$ does not depend explicitly on the Planck mass. Indeed, the gravitational constant (in natural units, $G = M_{Planck}^{-2}$) arising in the expressions of the early and late time vacuum energy densities cancels out in the above ratio. By assuming that $H_F \sim 10^{-42} \text{GeV}$ and $H_I \sim 10^{19} \text{GeV}$ (the cut-off of classical theory of gravity), one finds that the ratio $\frac{\rho_{\Lambda F}}{\rho_{\Lambda I}} \sim 10^{-122}$ suggested by quantum field theory. In this context, the open new perspective is related to the search of covariant action principle where both scales arise naturally. One possibility is related with models whose theoretical foundations are based on modified gravity theories ($F(R), F(T)$, etc).

- *de Sitter Instability and the future of the Universe*: Another interesting aspect associated with the presence of two extreme Sitter phases as discussed here, are the intrinsic instability of such spacetimes. Long time ago, Hawking shown that the spacetime of a static black hole is thermodynamically unstable to macroscopic fluctuation in the temperature of the horizon [19]. Later on, it was also demonstrated by Mottola [20] based on the validity of the generalized second law of thermodynamics that the same arguments used by Hawking in the case of black holes remain valid for the de Sitter spacetime. In the case of the primordial de Sitter phase, described here by the characteristic scale H_I , such an instability is dynamically described by solution (10) for $H(a)$. As we know, it behaves like a ‘repeller’ driving the model to the radiation phase. However, the instability result in principle must also be valid to the final de Sitter stage which behaves like an attractor. In this way, once the final de Sitter phase is reached, the spacetime would evolve to an energy scale smaller than H_F thereby starting a new evolutionary ‘cycle’ in the long run.

Finally, it should also be stressed that the solution presented here for the main cosmological puzzles is not an exclusive characteristic of this phenomenological decaying vacuum models evolving from de Sitter to de Sitter. For instance, it was recently proved that in the background level such models are equivalent to gravitationally induced particle production cosmologies [21, 22] by identifying $\Lambda(t) \equiv \rho\Gamma/3H$, where Γ is the gravitational particle production rate [23]. The dynamical equivalence with the present scenario at early and late times was discussed in Refs. [23, 24]. In principle, this means that

alternative scenarios evolving smoothly between two extreme de Sitter phases are also potentially able to provide viable solutions of the main cosmological puzzles with the proviso that in such models $\Omega_\Lambda \equiv 0$. Naturally, such alternatives to the decaying vacuum models is unable to shed some light on the cosmological constant problem since it becomes restricted to the realm of quantum field theory.

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